

MPI-Ph/95-11  
BONN-TH-95-04  
February 1995

**Rigid invariance as derived from BRS invariance**  
**The abelian Higgs model**

ELISABETH KRAUS\*  
*Physikalisches Institut*  
*Universität Bonn*  
*Nußallee 12, D-53115 Bonn*

KLAUS SIBOLD  
*Max-Planck-Institut für Physik*  
*Werner-Heisenberg-Institut*  
*Föhringer Ring 6, D-80805 München*

ABSTRACT

Consequences of a symmetry, e.g. relations amongst Green functions, are renormalization scheme independently expressed in terms of a rigid Ward identity. The corresponding local version yields information on the respective current. In the case of spontaneous breakdown one has to define the theory via the BRS invariance and thus to construct rigid and current Ward identity non-trivially in accordance with it. We performed this construction to all orders of perturbation theory in the abelian Higgs model as a prelude to the standard model. A technical tool of interest in itself is the use of a doublet of external scalar “background” fields. The Callan-Symanzik equation has an interesting form and follows easily once the rigid invariance is established.

---

\*Supported by DFG

## 1. Introduction

The precision of the next generation of experiments requires the calculation of two-loop contributions in the standard model. These calculations (for a recent review s. [1]) demand also higher precision as far as the more abstract side of the theory is concerned. Whereas at one-loop the compatibility of the renormalization scheme with the underlying theory need not be really discussed, at two loops this becomes much more urgent. Similar comments concern the rigid invariance. At one-loop the schemes used in practice are essentially compatible with the rigid invariance. In addition one has a more or less complete description available i.e. almost all divergent diagrams have been calculated, almost all interesting processes have been studied: there is not much need of a Ward identity (WI) which collects and formalizes the content and the consequences of rigid invariance. At two loops it will probably be impossible to perform systematic renormalization calculations without *explicit* use of the rigid invariance. It will serve at least as an indispensable tool for checking, but probably help even more in revealing the symmetry relation amongst Green functions and amplitudes. As an example where a specific WI was of great help one may look at [2], whereas in [3] a current algebra argument has been used. The latter arises from a local WI.

With this in mind we study in the present paper the rigid invariance in the abelian Higgs model, in a subsequent paper in the standard model. We formulate the rigid invariance in terms of a WI to all orders of perturbation theory. For these purposes external scalar “background” fields have to be introduced in order to absorb the breaking by the ‘t Hooft gauge fixing. It turns out, that in higher orders the classical WI is deformed in a well-defined way according to the normalization conditions, which one has imposed. Our presentation is essentially scheme independent and relies on the inductive construction order by order in the perturbation series. For this reason the classical approximation is treated extensively since already there the requirements must uniquely determine the desired quantities. Higher orders are then seen not to change dramatically the picture. Furthermore in the abelian model one is immediately able to derive a local WI.

As an application of rigid and local symmetry we construct the Callan-Symanzik (CS) equation. With its help it is possible to compute the asymptotic logarithms

of the Green functions, and to find their relations according to the symmetries in a simple way. In an appendix we have collected the propagators of the model for general values of the gauge parameters.

## 2. The construction of the model

The model comprises a vector field  $A_\mu$  and two scalar fields  $\varphi_1, \varphi_2$  interacting in such a way that  $U(1)$  gauge invariance is spontaneously broken. In conventional normalization we find that

$$\Gamma_{inv} = \int \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* (D^\mu \phi) + \frac{1}{2} m_H^2 \phi^* \phi - \frac{1}{2} \frac{m_H^2}{m^2} e^2 (\phi^* \phi)^2 \right) \quad (2.1)$$

where

$$\begin{aligned} F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu, & \phi &\equiv \frac{1}{\sqrt{2}}(\varphi_1 + v + i\varphi_2), \\ D_\mu \phi &\equiv \partial_\mu \phi - ie A_\mu \phi, \end{aligned} \quad (2.2)$$

is invariant under the transformations

$$\delta_\omega \phi = ie\omega \phi, \quad \delta_\omega A_\mu = \partial_\mu \omega. \quad (2.3)$$

Choosing

$$v = \frac{m}{e} \quad (2.4)$$

one can convince oneself that the vector field  $A_\mu$  has mass  $m$ , the field  $\varphi_1$  is the Higgs field with mass  $m_H$  and the field  $\varphi_2$  is the would-be Goldstone field eaten up by  $A_\mu$ . The vacuum expectation values of the scalar fields are zero:

$$\langle \varphi_1 \rangle = 0 = \langle \varphi_2 \rangle. \quad (2.5)$$

Since  $\Gamma_{inv}$  is invariant for  $x$ -dependent  $\omega$ , it is a fortiori invariant under transformations with constant  $\omega$  (“rigid” transformations).

For the calculation of Green functions and higher order corrections it is necessary to fix the gauge:

$$\Gamma_{g.f.} = \int \left( \frac{1}{2} \xi B^2 + B(\partial A + \xi_A m \varphi_2) \right) \quad (2.6)$$

$B$  is an invariant auxiliary field

$$\delta_\omega B = 0. \quad (2.7)$$

The 't Hooft term with  $\xi_A \neq 0$  has to be introduced in order to avoid a non-integrable infrared singularity in the  $\langle \varphi_2 \varphi_2 \rangle$  propagator. This 't Hooft type gauge fixing violates not only the local gauge invariance, but also the rigid symmetry non-trivially

$$\delta_\omega \Gamma_{\text{g.f.}} = \int (\omega \square B + \omega B \xi_A m(\varphi_1 + v)). \quad (2.8)$$

Hence it is unavoidable to translate local gauge transformations into BRS transformations by introducing the Faddeev-Popov ( $\phi\pi$ ) fields  $c, \bar{c}$

$$\begin{aligned} sA_\mu &= \partial_\mu c & s\bar{c} &= B \\ sc &= 0 & sB &= 0 \\ s\varphi_1 &= -ec\varphi_2 & s\varphi_2 &= ec(\varphi_1 + v) \end{aligned} \quad (2.9)$$

and to require BRS invariance instead of (broken) gauge invariance in order to define the theory

$$\Gamma_{cl} = \Gamma_{inv} + \Gamma_{\text{g.f.}} + \Gamma_{\phi\pi} + \Gamma_{\text{ext.f.}} \quad (2.10)$$

Here

$$\Gamma_{\phi\pi} = \int (-\bar{c} \square c - e\bar{c} \xi_A m(\varphi_1 + v)c) \quad (2.11)$$

and we have furthermore added an external field dependent part

$$\Gamma_{\text{ext.f.}} = \int (Y_1(-ec\varphi_2) + Y_2(ec(\varphi_1 + v))) \quad (2.12)$$

because the BRS transformations are non-linear in propagating (and interacting) fields, hence have to be defined in higher orders in a nontrivial way.

The invariance under BRS transformations can be expressed in terms of the vertex functional  $\Gamma$  as the Slavnov-Taylor identity (ST)

$$s(\Gamma) \equiv \int \left( \partial c \frac{\delta \Gamma}{\delta A} + B \frac{\delta \Gamma}{\delta \bar{c}} + \frac{\delta \Gamma}{\delta \underline{Y}} \cdot \frac{\delta \Gamma}{\delta \underline{\varphi}} \right) = 0. \quad (2.13)$$

$\Gamma_{cl}$  is the lowest order in the perturbative expansion of  $\Gamma$ , the tree approximation. In terms of  $Z$ , the generating functional of Green functions, the ST identity reads

$$sZ \equiv \int \left( \partial_\mu J^\mu \frac{\delta Z}{\delta J_c} - J_{\bar{c}} \frac{\delta Z}{\delta J_B} + J_{\underline{\varphi}} \frac{\delta Z}{\delta \underline{Y}} \right) = 0, \quad (2.14)$$

Here  $J_x$  with  $x = c, \bar{c}, B, \mu, \varphi$  denotes the sources for the respective fields. In addition to (2.13)  $\Gamma_{cl}$  solves the gauge condition

$$\frac{\delta\Gamma}{\delta B} = \xi B + \partial A + \xi_A m \varphi_2 \quad (2.15)$$

which can be imposed in this form to all orders of perturbation theory. The ghost equation of motion

$$\frac{\delta\Gamma}{\delta \bar{c}} + \xi_A m \frac{\delta\Gamma}{\delta Y_2} = -\square c \quad (2.16)$$

follows from (2.13) and (2.15).

The importance of the ST identity originates on the one hand from the fact that it permits to prove unitarity, i.e. the possibility of constructing a Hilbert space of physical states within which the scattering proceeds (for physical initial states). On the other hand – as alluded to above – it defines the model in question once multiplets have been chosen and normalization conditions have been specified. This is a renormalization scheme independent procedure and therefore unquestionable, whereas giving an action and its counterterms is not.

In order to see which normalization conditions are needed we present explicitly the general solution of the ST identity (2.13) and the gauge condition (2.15) in the tree approximation. It has the form

$$\Gamma_{cl}^{gen} = \int \left( \frac{1}{2} \xi B^2 + B(\partial A + \xi_A m \varphi_2) - \bar{c} \square c \right) + \hat{\Gamma} \quad (2.17)$$

$$\begin{aligned} \hat{\Gamma} = & \Lambda(A, \varphi_1, \varphi_2) + \int \hat{e}(-Y_1 z_2 \varphi_2 + Y_2 z_1(\varphi_1 + v) \\ & - \xi_A m \bar{c} z_1(\varphi_1 + v))c \end{aligned} \quad (2.17a)$$

$$\begin{aligned} \Lambda = & \int \left( -\frac{z_A}{4} F_{\mu\nu} F^{\mu\nu} + \frac{z_1}{2} \partial \varphi_1 \partial \varphi_1 + \frac{z_2}{2} \partial \varphi_2 \partial \varphi_2 \right. \\ & + \hat{e} z_1 z_2 (\partial \varphi_1 \varphi_2 - \partial \varphi_2 \varphi_1) A + \frac{1}{2} \hat{e}^2 z_1 z_2 (z_1 \varphi_1^2 + z_2 \varphi_2^2) A^2 \\ & + \frac{1}{2} \hat{e}^2 z_2 z_1^2 v^2 A^2 - z_1 z_2 \hat{e} v \partial \varphi_2 A + \hat{e}^2 z_1^2 z_2 v \varphi_1 A^2 \\ & \left. + \frac{1}{2} \mu^2 (z_1 \varphi_1^2 + 2 z_1 v \varphi_1 + z_2 \varphi_2^2) - \frac{1}{4} \hat{\lambda} (z_1 \varphi_1^2 + 2 z_1 v \varphi_1 + z_2 \varphi_2^2 + z_1 v^2)^2 \right) \end{aligned} \quad (2.17b)$$

For the derivation of (2.17) we have also imposed invariance under charge conjugation (see table for quantum numbers).

The wave function normalizations  $z_1, z_2$  and  $z_A$ , the masses of the vector and the Higgs particle, i.e. the parameters  $v, \mu, \hat{\lambda}$  and the coupling  $\hat{e}$  are not prescribed

Table: *Quantum numbers*

fields	$A_\mu$	$B$	$\tilde{\varphi}_1$	$\tilde{\varphi}_2$	$c$	$\bar{c}$	$Y_1$	$Y_2$	$q_1$
dim	1	2	1	1	0	2	3	3	1
charge conj.	–	–	+	–	–	–	+	–	+
$Q_{\phi\pi}$	0	0	0	0	+1	–1	–1	–1	+1

$$\tilde{\varphi} = \varphi, \hat{\varphi}$$

by the ST identity. They have to be fixed by appropriate normalization conditions to all orders.

In order to have a particle interpretation we shall fix the mass poles for the physical particles:

$$\Gamma_{\varphi_1\varphi_1}(p^2 = m_H^2) = 0 \quad (2.18a)$$

$$\Gamma^T(p^2 = m^2) = 0 \quad (2.18b)$$

for  $\Gamma_{\mu\nu} = \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Gamma^T + \frac{p_\mu p_\nu}{p^2}\Gamma^L$  and require vanishing vacuum expectation value for  $\varphi_1$ :

$$\langle \varphi_1 \rangle = 0 \quad (2.18c)$$

(2.18a–c) fixes the parameters  $v, \hat{\lambda}$  and  $\mu$ .

A complete on-shell version of the model is then, in analogy to the standard model [4, 5, 6], defined by fixing also their residues there. These normalization conditions determine  $z_1$  and  $z_A$ :

$$\partial_{p^2}\Gamma_{\varphi_1\varphi_1}(p^2 = m_H^2) = 1 \quad (2.18d)$$

$$\partial_{p^2}\Gamma^T(p^2 = m^2) = 1 \quad (2.18e)$$

Only the amplitude  $z_2$  of the field  $\varphi_2$  will be prescribed off-shell

$$\partial_{p^2}\Gamma_{\varphi_2\varphi_2}(p^2 = \kappa^2) = 1 \quad (2.18f)$$

and the coupling  $\hat{e}$  is also fixed at an arbitrary normalization momentum  $\kappa^2$

$$\partial_p\Gamma_{\varphi_1\varphi_2A}(p = p_{sym}(\kappa)) = ie \quad (2.18g)$$

The gauge parameter  $\xi_A$  is introduced by requiring

$$\Gamma_{c\bar{c}}(p^2 = m_{\text{ghost}}^2) = 0 \quad m_{\text{ghost}}^2 = \xi_A^{(o)} m^2 \quad (2.18h)$$

In higher orders the  $\hbar$ -contributions of  $\xi_A = \xi_A^{(o)} + O(\hbar)$  guarantee that indeed a pole is generated at  $\xi_A^{(o)} m^2$  for the ghost propagator.

The model is thus described as in the standard form in terms of the following physical parameters: vector mass  $m$ , Higgs mass  $m_H$ , charge  $e$ .

It is instructive (e.g. for the derivation of Callan-Symanzik and renormalization group equation) to go over from the general to the standard form by an explicit redefinition of fields and parameters:

$$\begin{aligned} \varphi_1^o &= \sqrt{z_1} \varphi_1 & Y_1^o &= \frac{1}{\sqrt{z_1}} Y_1 \\ \varphi_2^o &= \sqrt{z_2} \varphi_2 & Y_2^o &= \frac{1}{\sqrt{z_2}} Y_2 \\ A_\mu^o &= \sqrt{z_A} A_\mu & c^o &= \sqrt{z_A} c \\ B^o &= \frac{1}{\sqrt{z_A}} B & \bar{c}^o &= \frac{1}{\sqrt{z_A}} \bar{c} \\ v^o &= \sqrt{z_1} v & \mu^o &= \mu \\ \lambda^o &= \hat{\lambda} & \xi_A^o &= \frac{z_A}{\sqrt{z_2}} \xi_A \\ \xi^o &= z_A \xi & e^o &= \sqrt{\frac{z_1 z_2}{z_A}} \hat{e} \end{aligned} \quad (2.19)$$

In a scheme, where renormalized and bare quantities are distinguished this constitutes their relation. It can be shown [7] that ST identity (2.13), gauge condition (2.15), charge conjugation invariance and the normalization conditions (2.18) indeed uniquely define the model to all orders. This means that the Green functions of the model are finite, unambiguously specified and independent of the renormalization scheme one has used in the course of their calculation. We have displayed this construction of the model, in particular the general solution and the normalization conditions in such detail because they are needed for the derivation of the rigid invariance from BRS invariance.

### 3. Rigid invariance

A symmetry relates a priori unrelated Green functions and gives thus rise to observable consequences like grouping particles into multiplets or relations amongst scattering amplitudes. Usually these informations are contained in Ward-identities (WI) or in conservation equations for currents and charges. We were forced to define our model via the ST identity and have therefore now the task to derive these relations in accordance with the latter. In the present section we shall prove a linear WI, in the next section the current conservation equation.

#### 3.1. CLASSICAL APPROXIMATION

It is obvious that rigid invariance of  $\Gamma_{\text{inv}}$  (2.1) can be expressed by the WI

$$W\Gamma_{\text{inv}} \equiv \int \left( -\varphi_2 \frac{\delta}{\delta\varphi_1} + (\varphi_1 + v) \frac{\delta}{\delta\varphi_2} \right) \Gamma_{\text{inv}} = 0 \quad (3.1)$$

For  $\Gamma_{cl}$  (2.10) one finds immediately, that the WI is broken by the t'Hooft gauge fixing:

$$\begin{aligned} W\Gamma_{cl} &\equiv \int \left( -\varphi_2 \frac{\delta}{\delta\varphi_1} + (\varphi_1 + v) \frac{\delta}{\delta\varphi_2} - Y_2 \frac{\delta}{\delta Y_1} + Y_1 \frac{\delta}{\delta Y_2} \right) \Gamma_{cl} \\ &= \int (\xi_A m B(\varphi_1 + v) + \xi_A m e \bar{c} c \varphi_2) \\ &= s \int \xi_A m \bar{c} (\varphi_1 + v) \end{aligned} \quad (3.2)$$

here  $v = \frac{m}{e}$ . More interesting, namely pointing into the direction of our present problem, is the *postulate* requiring rigid invariance on the general classical action (2.17) up to the t'Hooft breaking:

$$W\Gamma_{cl}^{\text{gen}} = s \int \xi_A m \bar{c} (\varphi_1 + \frac{m}{e}) \quad (3.3)$$

because it enforces the relation

$$z_1 = z_2 \quad (3.4)$$

A rigid invariance as specified by (3.3) replaces one normalization condition, it relates the normalization of the wave function  $z_2$  to the normalization  $z_1$ . But if one wants to calculate Green functions in the on-shell normalization, one has to ensure the desired normalization by an explicit normalization condition. Therefore we allow an appropriate deformation of the WI operator rather than using the WI



as normalization condition, i.e. we stick to (2.18) and study the consequences for the WI.

In the tree approximation it can be read off from the ST identity for constant ghost fields that the following general WI holds

$$\begin{aligned}
W^{\text{gen}}\Gamma_{cl}^{\text{gen}} &\equiv z \int \left( -\sqrt{\frac{z_2}{z_1}} \varphi_2 \frac{\delta}{\delta \varphi_1} + \sqrt{\frac{z_1}{z_2}} (\varphi_1 + v) \frac{\delta}{\delta \varphi_2} \right. \\
&\quad \left. - \sqrt{\frac{z_1}{z_2}} Y_2 \frac{\delta}{\delta Y_1} + \sqrt{\frac{z_2}{z_1}} Y_1 \frac{\delta}{\delta Y_2} \right) \Gamma_{cl}^{\text{gen}} \\
&= z \sqrt{\frac{z_1}{z_2}} s \int \xi_A m \bar{c} (\varphi_1 + v)
\end{aligned} \tag{3.5}$$

(The factor  $z$  indicates that the overall normalization is arbitrary.)  $W^{\text{gen}}$  indeed qualifies for a WI operator in this abelian model: it is odd under charge conjugation. In this sense it is a deformed version of  $W$  in (3.2) and reduces to it in the tree approximation when the normalization conditions (2.18) are applied. The factors  $z_1, z_2$  will get their real content in higher orders, but serve here as indicator of what type of deformation is at least to be expected there.

### 3.2. CLASSICAL APPROXIMATION WITH EXTERNAL FIELDS

By now the calculations of rigid invariance (3.2), (3.5) have been carried out in the classical approximation where the vertex functional is a completely local object. In higher orders the possible forms of rigid invariance cannot be easily read off from the ST identity since the ghost fields  $c$  and  $\bar{c}$  interact, in particular appear in the internal lines of the loop corrections. For the same reason the r.h.s. of the WI – the breaking by the 't Hooft gauge fixing – becomes a true insertion and requires a non-trivial and unambiguous definition.

In order to proceed we couple the breaking of rigid invariance in the classical approximation to the action by introducing suitably transforming external fields [8, 9, 10]. The aim is to render the rigid WI homogeneous, hence a doublet  $\begin{pmatrix} \hat{\varphi}_1 \\ \hat{\varphi}_2 \end{pmatrix}$  of scalar external fields is appropriate, since the breaking transforms as a doublet. BRS invariance is not broken by the gauge fixing, so there is some freedom in choosing the BRS transformation properties of  $\hat{\varphi}$ . Since the breaking in the rigid WI is a BRS variation we can prescribe that  $\hat{\varphi}$  transforms under BRS as a doublet too

$$s \begin{pmatrix} \hat{\varphi}_1 \\ \hat{\varphi}_2 \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}. \tag{3.6}$$

This assignment will also turn out to be very natural when we study the gauge parameter dependence of the theory in an algebraic way. Under charge conjugation we require  $\hat{\varphi}_1, q_1$ , to be even,  $\hat{\varphi}_2, q_2$  to be odd. The (ultraviolet) dimension of all of them is taken to be 1.

The ST identity is enlarged to

$$s(\Gamma) \equiv \int \left( \partial_\mu c \frac{\delta \Gamma}{\delta A} + B \frac{\delta \Gamma}{\delta \bar{c}} + \frac{\delta \Gamma}{\delta \underline{Y}} \cdot \frac{\delta \Gamma}{\delta \underline{\varphi}} + \underline{q} \frac{\delta \Gamma}{\delta \underline{\hat{\varphi}}} \right) = 0 \quad (3.7)$$

and has to be solved in this generality. The general solution  $\Gamma_{cl}^{\text{gen}}$  can be decomposed quite analogously to (2.10) and (2.17) as

$$\Gamma_{cl}^{\text{gen}} = \Lambda(A, \bar{\varphi}_1, \bar{\varphi}_2) + \Gamma_{\text{g.f.}} + \Gamma_{\phi\pi} + \Gamma_{\text{ext.f.}} \quad (3.8)$$

where

$$\bar{\varphi}_1 \equiv \varphi_1 - x_1 \hat{\varphi}_1 \quad , \quad \bar{\varphi}_2 \equiv \varphi_2 - x_2 \hat{\varphi}_2. \quad (3.9)$$

$\Lambda$  is given in (2.17b), with  $\varphi \rightarrow \bar{\varphi}$ .

$$\begin{aligned} \Gamma_{\text{ext.f.}} = & \int (Y_1(-ez_2 \bar{\varphi}_2 c + x_1 q_1) \\ & + Y_2(ez_1(\bar{\varphi}_1 + v)c + x_2 q_2)) \end{aligned} \quad (3.10)$$

$x_{1,2}$  are new free parameters of the model. Therefore in addition to (2.18) we have to give normalization conditions for the parameters  $x_1$  and  $x_2$  which we choose on the external field part:

$$\Gamma_{Y_2 q_2}|_{p^2=\kappa^2} = x_2 \quad (3.11a)$$

$$\Gamma_{Y_1 q_1}|_{p^2=\kappa^2} = x_1 \quad (3.11b)$$

The gauge fixing terms are not restricted by the ST identity so we take a linear gauge in the propagating fields:

$$\Gamma_{\text{g.f.}} = \int \left( \frac{1}{2} \xi B^2 + B \partial A - e B \left( (\hat{\varphi}_1 - \xi_A \frac{m}{e}) \varphi_2 - \hat{\varphi}_2 (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \right) \right) \quad (3.12)$$

The contribution  $\xi_A m B \varphi_2$  was the starting point ('t Hooft gauge), its rigid variation is coupled to  $\hat{\varphi}_2$ , thus maintaining charge conjugation invariance; the term  $B \hat{\varphi}_1 \varphi_2$  is put in for having all linear terms; factors are chosen for later convenience. The parameter  $\xi_A$  is fixed by (2.18h),  $\hat{\xi}_A$  is a further free parameter, which is seen to be

fixed by the WI below (cf. (3.18)). Terms bilinear in the external fields are omitted because they will not be created by radiative corrections. To be more explicit: the gauge condition

$$\frac{\delta\Gamma}{\delta B} = \xi B + \partial A - e \left( \left( \hat{\varphi}_1 - \xi_A \frac{m}{e} \right) \varphi_2 - \hat{\varphi}_2 \left( \varphi_1 - \hat{\xi}_A \frac{m}{e} \right) \right) \quad (3.13)$$

can be postulated and integrated to yield  $\Gamma_{\text{g.f.}}$  in all orders. This means in particular that the normalization of the fields  $\hat{\varphi}_{1,2}$  is implicitly fixed by (3.12).

Once  $\Gamma_{\text{g.f.}}$  is given, the  $\phi\pi$ -part is prescribed by the ST identity:

$$\begin{aligned} \Gamma_{\phi\pi} = & \int \left( -\bar{c} \square c + e\bar{c}(q_1\varphi_2 - q_2(\varphi_1 - \hat{\xi}_A \frac{m}{e})) \right. \\ & + e\bar{c}(\hat{\varphi}_1 - \xi_A \frac{m}{e})(ez_1(\bar{\varphi}_1 + v)c + x_2q_2) \\ & \left. - e\bar{c}\hat{\varphi}_2(-ez_2\bar{\varphi}_2c + x_1q_1) \right) \end{aligned} \quad (3.14)$$

The ghost equ. of motion has the form

$$\begin{aligned} \frac{\delta\Gamma}{\delta\bar{c}} + e\bar{\varphi}_2 \frac{\delta\Gamma}{\delta Y_1} - e(\hat{\varphi}_1 - \xi_A \frac{m}{e}) \frac{\delta\Gamma}{\delta Y_2} \\ = -\square c + eq_1\varphi_2 - eq_2 \left( \varphi_1 - \hat{\xi}_A \frac{m}{e} \right) \end{aligned} \quad (3.15)$$

Returning now to the discussion of rigid invariance we first note that from  $\Lambda$  and  $\Gamma_{\text{ext.f.}}$  a separate transformation law for  $\varphi$  and  $\hat{\varphi}$  cannot yet be derived, since they only depend on the combination  $\bar{\varphi}$ . But when requiring a rigid invariance on  $\Gamma_{\text{g.f.}}$  and on  $\Gamma_{\phi\pi}$ , then separate transformation laws for  $\varphi$  and  $\hat{\varphi}$  emerge and one is led to the invariance of the general classical action (3.8) in the following form

$$\begin{aligned} W^{\text{gen}} \Gamma_{cl}^{\text{gen}} \\ \equiv \int \left( -\sqrt{\frac{z_2}{z_1}} \varphi_2 \frac{\delta}{\delta \varphi_1} + \sqrt{\frac{z_1}{z_2}} (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \frac{\delta}{\delta \varphi_2} - \sqrt{\frac{z_1}{z_2}} Y_2 \frac{\delta}{\delta Y_1} + \sqrt{\frac{z_2}{z_1}} Y_1 \frac{\delta}{\delta Y_2} \right. \\ \left. - \sqrt{\frac{z_2}{z_1}} \hat{\varphi}_2 \frac{\delta}{\delta \hat{\varphi}_1} + \sqrt{\frac{z_1}{z_2}} (\hat{\varphi}_1 - \xi_A \frac{m}{e}) \frac{\delta}{\delta \hat{\varphi}_2} - \sqrt{\frac{z_2}{z_1}} q_2 \frac{\delta}{\delta q_1} + \sqrt{\frac{z_1}{z_2}} q_1 \frac{\delta}{\delta q_2} \right) \Gamma_{cl}^{\text{gen}} = 0 \end{aligned} \quad (3.16)$$

This WI restricts the parameters  $x_1$  and  $x_2$ , in the classical approximation one has

$$x_1 = x_2 \equiv x. \quad (3.17)$$

Also the free parameter  $\hat{\xi}_A$  is determined by the WI (3.16) in terms of  $v, x$  and  $\xi_A$

$$-\hat{\xi}_A \frac{m}{e} = v - x \xi_A \frac{m}{e} \quad (3.18)$$

If one applies the normalization conditions on the classical action, then  $z_1 = z_2 = 1$ , but these factors indicate, how the classical transformations may be deformed in higher orders, if we fix  $z_1$  and  $z_2$  by independent normalization conditions as e.g. (2.18d) and (f). The tree value of  $\hat{\xi}_A^{(o)}$  is given by  $\hat{\xi}_A^{(o)} = -1 + x\xi_A$ .

It is easily verified that the WI (3.16) reproduces the breaking (3.2) at  $\hat{\varphi} = 0$  due to the inhomogeneous term  $-\sqrt{\frac{z_1}{z_2}}\xi_A \frac{m}{e} \frac{\delta}{\delta\hat{\varphi}_2}$  in the WI operator:

$$\begin{aligned} & \int \left( -\sqrt{\frac{z_2}{z_1}}\varphi_2 \frac{\delta}{\delta\varphi_1} + \sqrt{\frac{z_1}{z_2}}(\varphi_1 + v) \frac{\delta}{\delta\varphi_2} - \sqrt{\frac{z_1}{z_2}}Y_2 \frac{\delta}{\delta Y_1} + \sqrt{\frac{z_2}{z_1}}Y_1 \frac{\delta}{\delta Y_2} \right) \Gamma_{cl}^{\text{gen}} \Big|_{\hat{\varphi}=0=\underline{q}} \\ &= \sqrt{\frac{z_1}{z_2}}\xi_A \frac{m}{e} \int \left( \frac{\delta\Gamma_{cl}^{\text{gen}}}{\delta\hat{\varphi}_2} - x \frac{\delta\Gamma_{cl}^{\text{gen}}}{\delta\varphi_2} \right) \Big|_{\hat{\varphi}=0=\underline{q}} \\ &= \sqrt{\frac{z_1}{z_2}} \int (B(\varphi_1 + v) + z_2 e \bar{c} \varphi_2 c) \xi_A m \end{aligned} \tag{3.19}$$

This concludes our presentation of the classical approximation.

### 3.3. HIGHER ORDERS

The first task when looking into higher orders is to establish the enlarged ST identity (3.7) to all orders. As compared to (2.13) this requires solving the cohomology problem with doublet  $(\hat{\varphi}, \underline{q})$  included. We do not reproduce the respective calculations here but just note, that this cohomology is trivial hence (3.7) holds once suitable counterterms are admitted.

Our real aim is to demonstrate the validity of the deformed WI (3.16) to all orders, when acting on the generating functional of 1PI Green functions  $\Gamma$ . The most important ingredient for the proof is to note that  $W^{\text{gen}}$  has symmetry properties with respect to BRS invariance. This means the following: The action principle tells us that

$$W^{\text{gen}}\Gamma = \Delta \cdot \Gamma \tag{3.20}$$

where  $\Delta$  is a local integrated insertion (i.e. a sum of integrated field monomials) with dimension less than or equal to four, odd under charge conjugation and  $\phi\pi$ -charge zero. Acting now with  $W^{\text{gen}}$  on the ST identity (3.7) we obtain

$$0 = W^{\text{gen}} s(\Gamma) = s_\Gamma(W^{\text{gen}}\Gamma) = s_\Gamma(\Delta \cdot \Gamma) \tag{3.21a}$$

where

$$s_\Gamma \equiv \int \left( \partial_c \frac{\delta}{\delta A} + B \frac{\delta}{\delta \bar{c}} + \frac{\delta \Gamma}{\delta \underline{Y}} \cdot \frac{\delta}{\delta \underline{\varphi}} + \frac{\delta \Gamma}{\delta \underline{\varphi}} \cdot \frac{\delta}{\delta \underline{Y}} + \underline{q} \frac{\delta}{\delta \hat{\varphi}} \right). \quad (3.21b)$$

i.e.  $\Delta$  is BRS invariant. Hence we call the differential operator  $W^{\text{gen}}$  BRS symmetric.

We derive the validity of a rigid WI by induction starting from the tree approximation, where we have verified (cf. (3.16) with  $z_1 = z_2 = 1$ )

$$W\Gamma_{cl} = 0 \quad (3.22)$$

$W$  is the usual WI operator of rigid invariance as given in (3.2), including the external fields  $\hat{\varphi}_{1,2}$  and  $q_{1,2}$ . From (3.22) follows that  $\Delta$  is of order  $\hbar$

$$(W\Gamma)^{(\leq 1)} = \Delta^{(1)} \quad (3.23)$$

and therefore:

$$(s_\Gamma(\Delta \cdot \Gamma))^{(1)} = s_{\Gamma_{cl}} \Delta^{(1)} = 0, \quad (3.24)$$

(3.24) constitutes a consistency condition for  $\Delta$ . Solving it is again solving a cohomology problem, now in the sector defined by the quantum numbers of  $\Delta$  (charge conjugation:  $-$ ,  $Q_{\phi\pi} : 0$ ). It turns out that the cohomology is trivial i.e.

$$\Delta = s_{\Gamma_{cl}} \hat{\Delta} \quad (3.25)$$

( $s_{\Gamma_{cl}}$  is nilpotent:  $s_{\Gamma_{cl}} s_{\Gamma_{cl}} = 0$ ), hence the list of all  $\Delta$  is fairly short.

$$\begin{aligned} \{\Delta_i\} = s_{\Gamma_{cl}} \int & Y_2 \varphi_1, Y_1 \varphi_2, Y_2, Y_2 \hat{\varphi}_1, Y_1 \hat{\varphi}_2, \\ & \bar{c} \varphi_1, \bar{c} \hat{\varphi}_1, \bar{c} \varphi_1 \hat{\varphi}_1, \bar{c} \varphi_2 \hat{\varphi}_2, \\ & \bar{c}, \bar{c} \hat{\varphi}_1^2, \bar{c} \hat{\varphi}_2^2, \bar{c} \varphi_1^2, \bar{c} \varphi_2^2, \bar{c} A^2 \end{aligned} \quad (3.26)$$

A glance on the terms containing  $\bar{c}$  shows that those of the first line were “used” for the gauge fixing (3.12), whereas those of the second line were not “used”. This will soon be seen to be relevant for the coefficients with which they appear in (3.20). In order to determine them we rewrite the monomials as differential operators to the

extent to which this is possible. For some this is obvious:

$$\begin{aligned}
s_{\Gamma_{cl}} \int Y_2 \varphi_1 &= \int \left( \varphi_1 \frac{\delta \Gamma_{cl}}{\delta \varphi_2} - Y_2 \frac{\delta \Gamma_{cl}}{\delta Y_1} \right) \\
s_{\Gamma_{cl}} \int Y_1 \varphi_2 &= \int \left( \varphi_2 \frac{\delta \Gamma_{cl}}{\delta \varphi_1} - Y_1 \frac{\delta \Gamma_{cl}}{\delta Y_2} \right) \\
s_{\Gamma_{cl}} \int Y_2 &= \int \frac{\delta \Gamma_{cl}}{\delta \varphi_2} \\
s_{\Gamma_{cl}} \int Y_2 \hat{\varphi}_1 &= \int \left( \hat{\varphi}_1 \frac{\delta \Gamma_{cl}}{\delta \varphi_2} - Y_2 q_1 \right) \\
s_{\Gamma_{cl}} \int Y_1 \hat{\varphi}_2 &= \int \left( \hat{\varphi}_2 \frac{\delta \Gamma_{cl}}{\delta \varphi_1} - Y_1 q_2 \right)
\end{aligned} \tag{3.27}$$

For some others this rewriting requires a little calculation:

$$\begin{aligned}
\int \frac{\delta \Gamma_{cl}}{\delta \hat{\varphi}_2} &= s_{\Gamma_{cl}} \int (-x Y_2 + e \bar{c} (\varphi_1 - \hat{\xi}_A \frac{m}{e})) \\
\int \left( \hat{\varphi}_2 \frac{\delta}{\delta \hat{\varphi}_1} + q_2 \frac{\delta}{\delta q_1} \right) \Gamma_{cl} &= s_{\Gamma_{cl}} \int (-x \hat{\varphi}_2 Y_1 - e \bar{c} \hat{\varphi}_2 \varphi_2) \\
\int \left( \hat{\varphi}_1 \frac{\delta}{\delta \hat{\varphi}_2} + q_1 \frac{\delta}{\delta q_2} \right) \Gamma_{cl} &= s_{\Gamma_{cl}} \int \left( -x \hat{\varphi}_1 Y_2 + e \bar{c} \hat{\varphi}_1 (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \right)
\end{aligned} \tag{3.28}$$

From this representation it is also clear that these differential operators are symmetric with respect to BRS like  $W^{\text{gen}}$ , in fact they constitute just  $W^{\text{gen}}$ ! The validity of the classical WI (3.22) implies immediately that not all of the above differential operators are linearly independent when acting on  $\Gamma_{cl}$  and that consequently the polynomials in (3.26) are not independent. Hence we have to eliminate one of the polynomials in the first line of (3.26) e.g.  $s_{\Gamma_{cl}} \int (Y_1 \varphi_2)$ . Now we are able to rewrite (3.23) as follows

$$\begin{aligned}
(W\Gamma)^{(\leq 1)} &= \int \left( - (u_1^{(1)} \varphi_1 + v^{(1)}) \frac{\delta \Gamma}{\delta \varphi_2} + u_1^{(1)} Y_2 \frac{\delta \Gamma}{\delta Y_1} \right. \\
&\quad + u_3^{(1)} (\hat{\varphi}_2 \frac{\delta \Gamma}{\delta \hat{\varphi}_1} + q_2 \frac{\delta \Gamma}{\delta q_1}) \\
&\quad - (u_4^{(1)} \hat{\varphi}_1 + w^{(1)}) \frac{\delta \Gamma}{\delta \hat{\varphi}_2} - u_4^{(1)} q_1 \frac{\delta \Gamma}{\delta q_2} \Big) \\
&\quad + \left( u_5^{(1)} \left( \hat{\varphi}_1 \frac{\delta \Gamma}{\delta \varphi_2} - Y_2 q_1 \right) + u_6^{(1)} \left( \hat{\varphi}_2 \frac{\delta \Gamma}{\delta \varphi_1} - Y_1 q_2 \right) \right. \\
&\quad + s_{\Gamma_{cl}} \left( u_7^{(1)} \bar{c} + u_8^{(1)} \bar{c} \hat{\varphi}_1 + u_9^{(1)} \bar{c} \hat{\varphi}_1^2 + u_{10}^{(1)} \bar{c} \hat{\varphi}_2^2 \right. \\
&\quad \left. \left. + u_{11}^{(1)} \bar{c} \varphi_1^2 + u_{12}^{(1)} \bar{c} \varphi_2^2 + u_{13}^{(1)} \bar{c} A^2 \right) \right)
\end{aligned} \tag{3.29}$$

The test on the gauge fixing condition (3.13) leads to the following relations among the coefficients:

$$\begin{aligned} u_4^{(1)} = u_1^{(1)}, u_3^{(1)} = 0 \quad u_{11}^{(1)} = u_{12}^{(1)} = u_{13}^{(1)} = 0, \\ u_9^{(1)} = eu_5^{(1)} \quad u_{10}^{(1)} = -eu_6^{(1)} \end{aligned} \quad (3.30)$$

By appropriate choice of the shift parameter and by fixing thereby implicitly  $\hat{\xi}_A$  we can rewrite (3.29) in the following form ( $u_1^{(1)} \equiv u^{(1)}$ ):

$$\begin{aligned} ((W + \delta W^{(1)})\Gamma)^{(\leq 1)} &\equiv \int \left( -\varphi_2 \frac{\delta}{\delta \varphi_1} + ((1 + u^{(1)})(\varphi_1 - \hat{\xi}_A \frac{m}{e})) \frac{\delta}{\delta \varphi_2} \right. \\ &\quad + Y_1 \frac{\delta}{\delta Y_2} - (1 + u^{(1)})Y_2 \frac{\delta}{\delta Y_1} \\ &\quad - \hat{\varphi}_2 \frac{\delta}{\delta \hat{\varphi}_1} + ((1 + u^{(1)})(\hat{\varphi}_1 - \xi_A \frac{m}{e})) \frac{\delta}{\delta \hat{\varphi}_2} \\ &\quad \left. - q_2 \frac{\delta}{\delta q_1} + (1 + u^{(1)})q_1 \frac{\delta}{\delta q_2} \right) \Gamma \\ &= \int \left( u_5^{(1)} \left( \hat{\varphi}_1 \frac{\delta \Gamma_{cl}}{\delta \varphi_2} - Y_2 q_1 \right) + u_6^{(1)} \left( \hat{\varphi}_2 \frac{\delta \Gamma_{cl}}{\delta \varphi_1} - Y_1 q_2 \right) \right. \\ &\quad \left. + s_{\Gamma_{cl}} \left( -\xi_A m u_5^{(1)} \bar{c} \hat{\varphi}_1 + eu_5^{(1)} \bar{c} \hat{\varphi}_1^2 - eu_6^{(1)} \bar{c} \hat{\varphi}_2^2 \right) \right) \end{aligned} \quad (3.31)$$

Thereby we have taken all operators which appear already in the classical WI operator on the l.h.s. defining a deformed WI operator in 1-loop order

$$W_1 = W + \delta W^{(1)} \quad (3.32)$$

$W_1$  is quite analogous to the operator  $W^{\text{gen}}$  (3.16) of the classical approximation.

It remains to be shown, that the r.h.s. of (3.31) is actually vanishing in 1-loop order. Testing with respect to  $Y_2 q_1$  resp.  $Y_1 q_2$  yields equations for  $u_5^{(1)}$  and  $u_6^{(1)}$ :

$$u_5^{(1)} = -\Gamma_{Y_1 q_1}^{(1)} + \Gamma_{Y_2 q_2}^{(1)} - \hat{\xi}_A \frac{m}{e} \Gamma_{\varphi_2 Y_2 q_1}^{(1)} - \xi_A \frac{m}{e} \Gamma_{\hat{\varphi}_2 Y_2 q_1}^{(1)} \quad (3.33)$$

$$u_6^{(1)} = -\Gamma_{Y_1 q_1}^{(1)} + \Gamma_{Y_2 q_2}^{(1)} - \hat{\xi}_A \frac{m}{e} \Gamma_{\varphi_2 Y_1 q_2}^{(1)} - \xi_A \frac{m}{e} \Gamma_{\hat{\varphi}_2 Y_1 q_2}^{(1)} \quad (3.34)$$

The three-point-functions disappear in the limit of infinite momentum, hence

$$u_5^{(1)} = u_6^{(1)}. \quad (3.35)$$

The WI becomes

$$\begin{aligned}
(W_1\Gamma)^{(\leq 1)} &= u_5^{(1)} \int \left( \left( \hat{\varphi}_1 \frac{\delta\Gamma}{\delta\varphi_2} - Y_2 q_1 + \hat{\varphi}_2 \frac{\delta\Gamma}{\delta\varphi_1} - Y_1 q_2 \right) \right. \\
&\quad \left. + s_{\Gamma_{cl}} \left( -\xi_A m \bar{c} \hat{\varphi}_1 + e \bar{c} \hat{\varphi}_1^2 - e \bar{c} \hat{\varphi}_2^2 \right) \right) \\
&= u_5^{(1)} s_{\Gamma_{cl}} \int \left( Y_2 \hat{\varphi}_1 + Y_1 \hat{\varphi}_2 + \bar{c} (-\xi_A m \hat{\varphi}_1 + e \hat{\varphi}_1^2 - e \hat{\varphi}_2^2) \right).
\end{aligned} \tag{3.36}$$

The breaking is a variation with respect to the classical WI:

$$\begin{aligned}
(W_1\Gamma)^{(\leq 1)} &= u_5^{(1)} s_{\Gamma_{cl}} W \int (-Y_1 \hat{\varphi}_1 + \bar{c} \hat{\varphi}_1 \hat{\varphi}_2) \\
&= u_5^{(1)} W s_{\Gamma_{cl}} \int (-Y_1 \hat{\varphi}_1 + \bar{c} \hat{\varphi}_1 \hat{\varphi}_2)
\end{aligned} \tag{3.37}$$

Hence we are able to write the variation as a local counterterm to  $\Gamma$ , establishing thereby the deformed 1-loop WI:

$$W_1 \left( \Gamma - u_5^{(1)} s_{\Gamma} \int (-Y_1 \hat{\varphi}_1 + \bar{c} \hat{\varphi}_1 \hat{\varphi}_2) \right) = O(\hbar^2). \tag{3.38}$$

It remains only to be seen that the counterterm  $u_5 s_{\Gamma} f(\dots)$  can be added to  $\Gamma$  without spoiling the ST identity. But this is clear because the counterterm is a BRS invariant whose coefficient we can fix as we wish.

More explicitly it is the parameter  $x_2$  (3.11), which is equal to  $x_1$  in the tree approximation (3.17) and has to be adjusted also in 1-loop order. If we write

$$x_2 = x_1 + x_2^{(1)} \tag{3.39}$$

we can determine  $x_2^{(1)}$  as the solution of

$$\left( \Gamma_{Y_2 q_2} - \Gamma_{q_1 Y_1} - \frac{m}{e} (1 + u^{(1)}) (\hat{\xi}_A \Gamma_{\varphi_2 Y_1 q_2} - \xi_A \Gamma_{\hat{\varphi}_2 Y_1 q_2}) \right) \Big|_{p^2=\kappa^2} = 0 \tag{3.40}$$

i.e. in 1-loop (c.f. (3.18))

$$x_2^{(1)} = -\frac{m}{e} \left( (1 - x \xi_A) \Gamma_{\varphi_2 Y_1 q_2} - \xi_A \Gamma_{\hat{\varphi}_2 Y_1 q_2} \right) \Big|_{p^2=\kappa^2} \tag{3.41}$$

Then the rigid WI holds at the one-loop order.

In higher orders one proceeds by induction. One starts by assuming that a rigid Ward-identity of the following form is valid to n-loop order

$$(W_n \Gamma)^{(\leq n)} = 0 \tag{3.42}$$



where  $W_n$  is defined by the sum of the classical WI operator and higher orders deformations analogously to (3.31):

$$W_n = W + \sum_{i=1}^n \delta W^{(i)} \quad (3.43)$$

From there one concludes by the same reasoning as in 1-loop order that a deformed WI also holds at order  $n+1$ , if one adjusts the parameters  $\hat{\xi}_A$  and  $x_2$  order by order appropriately.

The final form of the WI valid to all orders

$$\begin{aligned} W_\infty \Gamma &= 0 \quad (3.44) \\ W_\infty &\equiv \int \left( -\varphi_2 \frac{\delta}{\delta \varphi_1} + (1+u)(\varphi_1 - \hat{\xi}_A \frac{m}{e}) \frac{\delta}{\delta \varphi_2} + Y_1 \frac{\delta}{\delta Y_2} - (1+u)Y_2 \frac{\delta}{\delta Y_1} \right. \\ &\quad \left. - \hat{\varphi}_2 \frac{\delta}{\delta \hat{\varphi}_1} + (1+u)(\hat{\varphi}_1 - \xi_A \frac{m}{e}) \frac{\delta}{\delta \hat{\varphi}_2} - q_2 \frac{\delta}{\delta q_1} + (1+u)q_1 \frac{\delta}{\delta q_2} \right) \end{aligned} \quad (3.44a)$$

can be immediately compared with the general deformed WI operator  $W^{\text{gen}}$  (3.16) of the classical approximation: We see that the deformation is indeed as the general classical solution suggested.

$$W_\infty = z W^{\text{gen}} \quad (3.45)$$

Multiplying (3.16) with  $z = \sqrt{z_1/z_2}$  we can identify the deformation as

$$\frac{z_1}{z_2} = \frac{1 + \hat{z}_1}{1 + \hat{z}_2} = 1 + u \quad (3.46)$$

i.e. in 1-loop order

$$\hat{z}_1^{(1)} - \hat{z}_2^{(1)} = u^{(1)} \quad (3.47)$$

This combination of the wave function renormalizations is thus independent of how one removes the infinities, but depends only on the prescribed normalization.

#### 4. The local Ward-identity

In the *abelian* Higgs model one can construct from the deformed rigid WI (3.44) a local WI, which expresses invariance of the Green functions under deformed local gauge transformations. In contrast to QED this gauge invariance does not characterize the spontaneously broken model, but has to be derived from the Slavnov-Taylor identity.

We define the local WI operator from the rigid one, (3.16) and (3.44, 45), by taking away the integration:

$$\begin{aligned}
W^{\text{gen}} &\equiv \int dx w^{\text{gen}}(x) \\
w^{\text{gen}}(x) &\equiv \\
& -\sqrt{\frac{z_2}{z_1}} \varphi_2 \frac{\delta}{\delta \varphi_1} + \sqrt{\frac{z_1}{z_2}} (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \frac{\delta}{\delta \varphi_2} - \sqrt{\frac{z_1}{z_2}} Y_2 \frac{\delta}{\delta Y_1} + \sqrt{\frac{z_2}{z_1}} Y_1 \frac{\delta}{\delta Y_2} \\
& -\sqrt{\frac{z_2}{z_1}} \hat{\varphi}_2 \frac{\delta}{\delta \hat{\varphi}_1} + \sqrt{\frac{z_1}{z_2}} (\hat{\varphi}_1 - \xi_A \frac{m}{e}) \frac{\delta}{\delta \hat{\varphi}_2} - \sqrt{\frac{z_2}{z_1}} q_2 \frac{\delta}{\delta q_1} + \sqrt{\frac{z_1}{z_2}} q_1 \frac{\delta}{\delta q_2}
\end{aligned} \tag{4.1a}$$

In analogy to the treatment of the rigid WI we first want to study the application of the local WI operator on the general classical action (3.8). One verifies immediately

$$\left( \hat{e} \sqrt{z_1 z_2} w^{\text{gen}}(x) - \partial \frac{\delta}{\delta A} \right) \Gamma_{cl}^{\text{gen}} = \square B. \tag{4.2}$$

Using the normalization conditions we find in the tree approximation

$$\left( ew(x) - \partial \frac{\delta}{\delta A} \right) \Gamma_{cl} = \square B. \tag{4.3}$$

where  $w(x)$  is the original undeformed local WI operator (cf. (3.22)).

In order to proceed to higher orders we have again to classify the operators according to their transformation properties with respect to BRS and according to their quantum numbers. From the WI (3.44)

$$W^{\text{gen}} \Gamma = 0 \tag{4.4}$$

it follows with the help of the action principle, that

$$w^{\text{gen}}(x) \Gamma = [\partial^\mu j_\mu]_4 \cdot \Gamma \tag{4.5}$$

where the insertion  $\partial^\mu j_\mu$  is a total derivative which has dimension four,  $\phi\pi$ -charge zero and is odd under charge conjugation. Like  $W^{\text{gen}}$  the local  $w^{\text{gen}}(x)$  is BRS symmetric and  $\partial^\mu j_\mu$  is therefore a BRS invariant (cf. (3.21a, b))

$$0 = w^{\text{gen}}(x)s(\Gamma) = s_\Gamma([\partial j] \cdot \Gamma) \quad (4.6)$$

The same characterization is true for the operator  $\partial^\mu \frac{\delta\Gamma}{\delta A^\mu}$ . This is most easily seen by differentiating the ST identity (3.7)

$$s(\Gamma) \equiv \int \partial c \frac{\delta\Gamma}{\delta A} + B \frac{\delta\Gamma}{\delta \bar{c}} + \frac{\delta\Gamma}{\delta \underline{Y}} \cdot \frac{\delta\Gamma}{\delta \underline{\varphi}} + \underline{q} \frac{\delta\Gamma}{\delta \underline{\hat{\varphi}}} = 0 \quad (4.7)$$

with respect to  $c$ . From there we obtain

$$-\partial \frac{\delta\Gamma}{\delta A} - s_\Gamma\left(\frac{\delta\Gamma}{\delta c}\right) = 0, \quad (4.8)$$

i.e. it is not only a BRS invariant, but moreover a BRS variation. Proceeding now order by order we get in 1-loop

$$\left(ew^{\text{gen}}(x)\Gamma - \partial \frac{\delta\Gamma}{\delta A}\right)^{(\leq 1)} = \partial^\mu j_\mu^{(1)} \quad (4.9)$$

Following the discussion above a short calculation shows, that a basis for  $\partial^\mu j_\mu^{(1)}$  is given by the two terms  $\square B$  and  $\partial^\mu j_\mu^{\text{matter}}$ , where we define (recall  $\bar{\varphi}_i = \varphi_i - x\hat{\varphi}_i$ )

$$\begin{aligned} j_\mu^{\text{matter}} &= \bar{\varphi}_2 \partial_\mu \bar{\varphi}_1 - (\bar{\varphi}_1 + \frac{m}{e}) \partial_\mu \bar{\varphi}_2 \\ &\quad + eA_\mu (\bar{\varphi}_2^2 + (\bar{\varphi}_1 + \frac{m}{e})^2) \end{aligned} \quad (4.10)$$

This field polynomial is indeed invariant under  $s_{\Gamma_{cl}}$ . One of the two basis elements can be replaced by the operator  $\partial \frac{\delta\Gamma_{cl}}{\delta A}$  or  $w(x)\Gamma_{cl}$  respectively:

$$\begin{aligned} \partial \frac{\delta\Gamma_{cl}}{\delta A} &= ew(x)\Gamma_{cl} - \square B \\ &= e\partial^\mu j_\mu^{\text{matter}} - \square B \end{aligned} \quad (4.11)$$

Therefore we rewrite (4.9) into the following form:

$$\left(ew^{\text{gen}}(x)\Gamma - \partial \frac{\delta\Gamma}{\delta A}\right)^{(\leq 1)} = a^{(1)}w(x)\Gamma_{cl} + \square B \quad (4.12)$$

The coefficient of  $\square B$  can be determined by testing on the gauge fixing condition. Shifting the variation  $a^{(1)}w(x)\Gamma_{cl}$  from the r.h. to the l.h.s. one gets the 1-loop local

WI. Proceeding in the same way by induction we derive to all orders in perturbation theory a local WI, which involves the deformed operator  $w^{\text{gen}}$

$$\left(e(1+a)w^{\text{gen}}(x) - \partial \frac{\delta}{\delta A}\right) \Gamma = \square B \quad (4.13)$$

( $a = O(\hbar)$ ). When written in terms of  $\Gamma$ , the WI is most useful for the purposes of renormalization, but for other applications its formulation on  $Z$ , the generating functional for general Green functions, is also interesting. We therefore present this version too.

$$\left(e(1+a)w^{\text{gen}}[J](x) + i\partial^\mu J_\mu\right) Z[J] = \square \frac{\partial Z}{\partial J_B} \quad (4.14)$$

$$\begin{aligned} w^{\text{gen}}[J] \equiv & \quad (4.14a) \\ & \sqrt{\frac{z_2}{z_1}} J_{\varphi_1} \frac{\delta}{\delta J_{\varphi_2}} - \sqrt{\frac{z_1}{z_2}} J_{\varphi_2} \left(-i\hat{\xi}_A \frac{m}{e} + \frac{\delta}{\delta J_{\varphi_1}}\right) - \sqrt{\frac{z_1}{z_2}} Y_2 \frac{\delta}{\delta Y_1} + \sqrt{\frac{z_2}{z_1}} Y_1 \frac{\delta}{\delta Y_2} \\ & - \sqrt{\frac{z_2}{z_1}} \hat{\varphi}_2 \frac{\delta}{\delta \hat{\varphi}_1} + \sqrt{\frac{z_1}{z_2}} \left(\hat{\varphi}_1 - \xi_A \frac{m}{e}\right) \frac{\delta}{\delta \hat{\varphi}_2} - \sqrt{\frac{z_2}{z_1}} q_2 \frac{\delta}{\delta q_1} + \sqrt{\frac{z_1}{z_2}} q_1 \frac{\delta}{\delta q_2} \end{aligned}$$

It is important to note that this WI does not characterize the theory since it says nothing about the behaviour of the  $\phi\pi$ -ghosts and does not permit to conclude that  $\partial A$  is a free field. The inhomogeneous contributions in  $w(x)$  prohibit this conclusion. I.e. unlike the unbroken case one *has* to use the ST identity for the proof of unitarity and for a scheme independent characterization of the model. The local WI on the other hand permits one to study the fate of  $\partial A$  when inserted in Green functions as being closely related to the divergence of the current.

## 5. The Callan-Symanzik equation

The Callan-Symanzik (CS) equation describes the response of the system to scaling of all independent parameters carrying dimension of mass. In the present context of rigid invariance this is of special interest, because consequences of the underlying symmetry manifest themselves most clearly as relations of different coefficient functions.

### 5.1. THE CLASSICAL APPROXIMATION

In the classical approximation scale invariance is broken

$$\underline{m}\partial_{\underline{m}}\Gamma_{cl} \equiv (m\partial_m + m_H\partial_{m_H} + \kappa\partial_\kappa)\Gamma_{cl} = \Delta_m \quad (5.1)$$

by all terms in the classical action with dimension less than or equal to three. We can immediately calculate  $\Delta_m$  and find:

$$\begin{aligned} \Delta_m = & m^2 A^2 - m_H^2 \bar{\varphi}_1^2 \\ & - m\partial\bar{\varphi}_2 A + em\bar{\varphi}_1 A^2 - \frac{1}{2}\frac{m_H^2}{m}e\bar{\varphi}_1(\bar{\varphi}_1^2 + \bar{\varphi}_2^2) \\ & + ms(\bar{c}(\hat{\xi}_A\varphi_2 - \xi_A\hat{\varphi}_2)) \end{aligned} \quad (5.2)$$

where  $\bar{\varphi}_i = \varphi_i - x\hat{\varphi}_i$  (3.9) and  $\hat{\xi}_A = -1 + x\xi_A$  in the tree approximation (3.18). It is obvious that  $\Delta_m$  is even under charge conjugation. As a consequence of the BRS invariance of the theory it is in addition BRS invariant as can be seen more abstractly by applying  $\underline{m}\partial_{\underline{m}}$  to the ST identity (3.7)

$$0 = \underline{m}\partial_{\underline{m}}s(\Gamma_{cl}) = s_{\Gamma_{cl}}(\underline{m}\partial_{\underline{m}}\Gamma_{cl}) = s_{\Gamma_{cl}}(\Delta_m) \quad (5.3)$$

with  $s_{\Gamma}$  given in (3.21b). Furthermore it has a certain covariance with respect to the WI operator of rigid symmetry  $W$ (3.22).

$$[W, \underline{m}\partial_{\underline{m}}]\Gamma_{cl} = W\Delta_m = \hat{\xi}_A \frac{m}{e} \int \frac{\delta\Gamma_{cl}}{\delta\varphi_2} + \xi_A \frac{m}{e} \int \frac{\delta\Gamma_{cl}}{\delta\hat{\varphi}_2} \quad (5.4)$$

We have to check now, whether  $\Delta_m$  is uniquely determined by these characteristics. Because, if so, we can proceed this way to all orders, where the evaluation of  $\Delta_m$  is not possible with explicit coefficients. This analysis also gives a complete classification of the 2 and 3 dimensional insertions appearing in the breaking of scale invariance.

The BRS invariant terms contributing to  $\Delta_m$  are quickly listed. There is one invariant which is not a variation, the other terms are variations:

$$\int(\bar{\varphi}_1^2 + 2v\bar{\varphi}_1 + \bar{\varphi}_2^2), s_{\Gamma_{cl}} \int Y_1, s_{\Gamma_{cl}} \int \bar{c}\varphi_2, s_{\Gamma_{cl}} \int \bar{c}\hat{\varphi}_2 \quad (5.5)$$

Since in higher orders it is preferable to deal with differential operators instead of insertions, we replace  $s_{\Gamma_{cl}} \int \bar{c}\varphi_2$  by  $\int \frac{\delta\Gamma_{cl}}{\delta\hat{\varphi}_1}$  which according to

$$\frac{\delta\Gamma_{cl}}{\delta\hat{\varphi}_1} = x \frac{\delta\Gamma_{cl}}{\delta\varphi_1} - xe s_{\Gamma_{cl}}(\bar{c}\hat{\varphi}_2) - e s_{\Gamma_{cl}}(\bar{c}\varphi_2) \quad (5.6)$$

is also a variation. Hence  $\Delta_m$  can be represented in the form

$$\Delta_m = u_{inv} \int (\bar{\varphi}_1^2 + 2\bar{v}\bar{\varphi}_1 + \bar{\varphi}_2^2) + \int \left( u_1 \frac{\delta \Gamma_{cl}}{\delta \varphi_1} + u_2 \frac{\delta \Gamma_{cl}}{\delta \hat{\varphi}_1} + u_3 (B\hat{\varphi}_2 - \bar{c}q_2) \right) \quad (5.7)$$

Testing first on the gauge condition with respect to  $B$  (3.13) we find

$$u_3 + eu_1 = -\hat{\xi}_A m \quad u_2 = -\xi_A \frac{m}{e} \quad (5.8)$$

or

$$\begin{aligned} \underline{m} \partial_{\underline{m}} \Gamma_{cl} = & u_1 \int \frac{\delta \Gamma_{cl}}{\delta \varphi_1} - \xi_A \frac{m}{e} \int \frac{\delta \Gamma_{cl}}{\delta \hat{\varphi}_1} - (\hat{\xi}_A m + eu_1) \int (B\hat{\varphi}_2 - \bar{c}q_2) \\ & + u_{inv} \int (\bar{\varphi}_1^2 + 2\bar{v}\bar{\varphi}_1 + \bar{\varphi}_2^2) \end{aligned} \quad (5.9)$$

So far we can get with BRS invariance alone. But even without all external fields the decomposition into the variation  $s_{\Gamma_{cl}} Y_1$  (first term) and the non-variation (last term) would not be unique. At this point rigid invariance has to be used:  $\Gamma_{cl}$  satisfies the rigid WI (3.22) and according to (5.4) we extend  $\underline{m} \partial_{\underline{m}}$  to  $\underline{m} \tilde{\partial}_{\underline{m}}$  which by definition commutes with  $W$ :

$$\underline{m} \tilde{\partial}_{\underline{m}} \equiv \underline{m} \partial_{\underline{m}} + \hat{\xi}_A \frac{m}{e} \int \frac{\delta}{\delta \varphi_1} + \xi_A \frac{m}{e} \int \frac{\delta}{\delta \hat{\varphi}_1} \quad [W, \underline{m} \tilde{\partial}_{\underline{m}}] = 0. \quad (5.10)$$

Hence we identify

$$u_1 = -\hat{\xi}_A \frac{m}{e} \quad (5.11)$$

and rewrite (5.9) into the symmetric form

$$\underline{m} \tilde{\partial}_{\underline{m}} \Gamma_{cl} = u_{inv} \int (\bar{\varphi}_1^2 + 2\bar{v}\bar{\varphi}_1 + \bar{\varphi}_2^2) \quad (5.12)$$

The coefficient  $u_{inv}$  cannot be determined by symmetry considerations; one can calculate it by testing with respect to  $\varphi_1$

$$u_{inv} = \frac{1}{2} m_H^2 \quad (5.13)$$

Introducing a further external field  $\hat{\varphi}_0$  of dimension 2, even under charge conjugation, invariant under BRS and rigid transformations coupled to the invariant  $\bar{\varphi}_1^2 + 2\bar{v}\bar{\varphi}_1 + \bar{\varphi}_2^2$  we can finally write the CS equ. in the classical approximation, where  $\hat{\xi}_A = -1 + x\xi_A$ , as

$$\underline{m} \partial_{\underline{m}} \Gamma_{cl} = \frac{m}{e} (1 - x\xi_A) \int \frac{\delta \Gamma_{cl}}{\delta \varphi_1} - \xi_A \frac{m}{e} \int \frac{\delta \Gamma_{cl}}{\delta \hat{\varphi}_1} + \frac{1}{2} m_H^2 \int \frac{\delta \Gamma_{cl}}{\delta \hat{\varphi}_0} \quad (5.14)$$

One verifies immediately that (5.14) coincides with the explicit determination of  $\Delta_m$  (5.2).

Summarizing the result of these considerations in the tree approximation we can state that the r.h.s. of the CS equ. (5.14) is unique once we require invariance under BRS, rigid transformations and charge conjugation and limit its dimension by three.

## 5.2. HIGHER ORDERS

We shall try to follow closely the reasoning of the tree approximation. The action principle tells us that

$$\underline{m}\partial_{\underline{m}}\Gamma = \Delta_m \cdot \Gamma \quad (5.15)$$

where  $\Delta_m$  is now an insertion of power counting four, even under charge conjugation and still BRS invariant due to

$$0 = \underline{m}\partial_{\underline{m}}s(\Gamma) = s_{\Gamma}(\underline{m}\partial_{\underline{m}}\Gamma) = s_{\Gamma}(\Delta_m \cdot \Gamma). \quad (5.16)$$

Hence we have to extend the above list of BRS invariant insertions (5.5, 6) by those of dimension four, which we immediately give in the form of BRS-symmetric operators. For the following operators the correspondence to the BRS-symmetric insertions is again obvious

$$\begin{aligned} s_{\Gamma} \int \varphi_1 Y_1 &= \int (\varphi_1 \frac{\delta}{\delta \varphi_1} - Y_1 \frac{\delta}{\delta Y_1}) \Gamma & s_{\Gamma} \int \hat{\varphi}_1 Y_1 &= \int (\hat{\varphi}_1 \frac{\delta \Gamma}{\delta \varphi_1} - Y_1 q_1) \\ s_{\Gamma} \int \varphi_2 Y_2 &= \int (\varphi_2 \frac{\delta}{\delta \varphi_2} - Y_2 \frac{\delta}{\delta Y_2}) \Gamma & s_{\Gamma} \int \hat{\varphi}_2 Y_2 &= \int (\hat{\varphi}_2 \frac{\delta \Gamma}{\delta \varphi_2} - Y_2 q_2) \end{aligned} \quad (5.17a)$$

whereas it requires a short calculation to prove the independence of the further operators, when acting on  $\Gamma$ :

$$\int (A \frac{\delta}{\delta A} + c \frac{\delta}{\delta c}), \int (B \frac{\delta}{\delta B} + \bar{c} \frac{\delta}{\delta \bar{c}}), \int (\hat{\varphi}_i \frac{\delta}{\delta \hat{\varphi}_i} + q_i \frac{\delta}{\delta q_i}) \Gamma \quad (5.17b)$$

$$m_H \partial_{m_H}, e \partial_e, \xi \partial_{\xi} \Gamma. \quad (5.17c)$$

The insertion

$$\int (B \hat{\varphi}_1 \hat{\varphi}_2 - \bar{c} q_1 \hat{\varphi}_2 - \bar{c} \hat{\varphi}_1 q_2) \quad (5.17d)$$

cannot be replaced by a BRS symmetric operator. Together with (5.5) the insertions defined by (5.17) constitute a complete basis of BRS invariant insertions building

up  $\Delta_m$ . As one can easily convince oneself, the BRS symmetric operator  $\partial_x \Gamma$  is not independent, because

$$\begin{aligned} \partial_x \Gamma_{cl} = & - \int \left( \hat{\varphi}_1 \frac{\delta \Gamma_{cl}}{\delta \varphi_1} + \hat{\varphi}_2 \frac{\delta \Gamma_{cl}}{\delta \varphi_2} + Y_1 q_1 + Y_2 q_2 \right. \\ & \left. - e \xi_A m s_{\Gamma_{cl}}(\bar{c} \hat{\varphi}_2) \right) \quad \text{if } x \neq 0 \end{aligned} \quad (5.18)$$

The limit of  $x = 0$ , which is an allowed normalization, is not appropriately represented by  $\partial_x \Gamma$ , and therefore we will remain with (5.17).

The tree approximation taught us the lesson that we should make use of the rigid invariance if we want to construct a unique r.h.s. of the CS equ. Similarly to (5.16) the insertion  $\Delta_m$  also has a certain covariance with respect to  $W^{\text{gen}}$  (3.44, 45)

$$[W^{\text{gen}}, \underline{m} \partial_{\underline{m}}] = W^{\text{gen}}(\Delta_m \cdot \Gamma) = \sqrt{\frac{z_1}{z_2}} \frac{m}{e} \int \left( \hat{\xi}_A \frac{\delta \Gamma}{\delta \varphi_2} + \xi_A \frac{\delta \Gamma}{\delta \hat{\varphi}_2} \right) \quad (5.19)$$

which is the generalization of (5.4) to higher orders. Hence according to the derivation above we will consider instead of  $\underline{m} \partial_{\underline{m}}$  the symmetric version  $\underline{m} \tilde{\partial}_{\underline{m}}$  defined in (5.10), which can be shown to commute with  $W^{\text{gen}}$ , too, and the insertion  $\tilde{\Delta}_m \cdot \Gamma$

$$\underline{m} \tilde{\partial}_{\underline{m}} \Gamma = \tilde{\Delta}_m \cdot \Gamma \quad (5.20)$$

is  $W^{\text{gen}}$  symmetric. Therefore we symmetrize first of all the operators listed in (5.17) with respect to the general rigid WI operator  $W^{\text{gen}}$  (3.44). The leg counting operators amongst those of (5.17) are easily symmetrized. They read

$$\begin{aligned} \mathcal{N}_s &\equiv N_s - \hat{\xi}_A \frac{m}{e} \int \delta \varphi_1 \equiv \int \left( (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \delta \varphi_1 + \varphi_2 \delta \varphi_2 - Y_1 \delta Y_1 - Y_2 \delta Y_2 \right) \\ \hat{\mathcal{N}}_s &\equiv \hat{N}_s - \xi_A \frac{m}{e} \int \delta \hat{\varphi}_1 \equiv \int \left( (\hat{\varphi}_1 - \xi_A \frac{m}{e}) \delta \hat{\varphi}_1 + \hat{\varphi}_2 \delta \hat{\varphi}_2 + q_1 \delta q_1 + q_2 \delta q_2 \right) \\ N_A &\equiv \int \left( A \delta A + c \delta c \right) \\ N_B &\equiv \int \left( B \delta B + \bar{c} \delta \bar{c} \right) \end{aligned} \quad (5.21)$$

The mixed operators  $\hat{\varphi}_i \frac{\delta}{\delta \varphi_i}$  are symmetrized like the leg counting operators

$$\bar{\mathcal{N}}_s \equiv \bar{N}_s - \xi_A \frac{m}{e} \int \delta \varphi_1 \equiv \int \left( (\hat{\varphi}_1 - \xi_A \frac{m}{e}) \delta \varphi_1 + \hat{\varphi}_2 \delta \varphi_2 \right) \quad (5.22)$$

and the insertion

$$\bar{\mathcal{N}}_s \Gamma + \int (q_1 Y_1 + q_2 Y_2) \quad (5.23)$$



is BRS and  $W^{\text{gen}}$  invariant. Slightly more involved is the symmetrization of the differential operators in (5.17c)  $\nabla = m_H \partial_{m_H}, e \partial_e, \xi \partial_\xi$ . Their symmetrized extensions  $\tilde{\nabla}$  have the explicit form

$$\begin{aligned} \tilde{\nabla} \equiv & \nabla + \int \left( \nabla(\hat{\xi}_A \frac{m}{e}) \delta_{\varphi_1} + \nabla(\xi_A \frac{m}{e}) \delta_{\hat{\varphi}_1} \right) \\ & - \frac{1}{2} \nabla(\ln \frac{z_1}{z_2}) \int \left( (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \delta_{\varphi_1} - Y_1 \delta_{Y_1} \right. \\ & \left. + (\hat{\varphi}_1 - \xi_A \frac{m}{e}) \delta_{\hat{\varphi}_1} + q_1 \delta_{q_1} \right) \end{aligned} \quad (5.24)$$

The insertion (5.17d) is not invariant under rigid transformations and can therefore not contribute on the r.h.s. of (5.20). When inspecting the operators of dimension three (compare (5.7) and (5.14)) we find that only  $\int \frac{\delta}{\delta \varphi_0}$  can contribute  $W$ -symmetrically.

The basis of BRS and rigidly invariant differential operators which are charge conjugation invariant and have dimension less than or equal to four is thus provided by (5.21, 23, 24) and  $\int \delta \varphi_0$ . The insertion  $\tilde{\Delta}_m \cdot \Gamma$  (5.20) can therefore be decomposed in the following way

$$\begin{aligned} \tilde{\mathcal{C}}\Gamma \equiv & \left( \underline{m} \tilde{\partial}_{\underline{m}} + \beta_e e \tilde{\partial}_e + \beta_{m_H} m_H \tilde{\partial}_{m_H} + \beta_\xi \xi \tilde{\partial}_\xi \right. \\ & \left. - \gamma_s \mathcal{N}_s - \hat{\gamma}_s \hat{\mathcal{N}}_s - \bar{\gamma}_s \bar{\mathcal{N}}_s - \gamma_A \mathcal{N}_A - \gamma_B \mathcal{N}_B - \alpha_{\text{inv}} \int \delta \varphi_0 \right) \Gamma \\ = & \bar{\gamma}_s \int (q_1 Y_1 + q_2 Y_2) \end{aligned} \quad (5.25)$$

This is the CS-equ. in manifestly  $W$ -symmetric form. Its most important feature is the appearance of the  $\beta$ -function  $\beta_{m_H}$  which is a consequence of the physical normalization of the model. Since the scalar self-coupling  $\lambda$  is then not an independent parameter  $\beta_{m_H}$  has to replace  $\beta_\lambda$ . It is quite unclear, how in higher orders a formulation in terms of unphysical parameters could be related to the one in physical parameters (cf. [11, 12]).

Some additional information on the coefficients comes from testing on the gauge condition (3.13):

$$\gamma_B = -\gamma_A \quad (5.26a)$$

$$\beta_\xi = -2\gamma_A \quad (5.26b)$$

$$\beta_e + \gamma_A - \gamma_s - \hat{\gamma}_s = \frac{1}{2} (\beta_e e \partial_e + \beta_{m_H} m_H \partial_{m_H} + \beta_\xi \xi \partial_\xi) \ln \frac{z_1}{z_2} \quad (5.26c)$$

One has to note, that the last relation constitutes a connection of the anomalous dimension of the external fields with the coefficient functions of the propagating fields, but there is no similar relation for  $\bar{\gamma}_s$ .

One further relation emerges from the validity of the local WI (4.13)

$$\left(e\hat{a}w^{\text{gen}}(x) - \partial\frac{\delta}{\delta A}\right)\Gamma = \square B \quad \text{with} \quad \hat{a} = 1 + a \quad (5.27)$$

We calculate the commutators

$$\begin{aligned} [\tilde{\mathcal{C}}, e\hat{a}w^{\text{gen}}(x)] &= e\left(\beta_e\hat{a} + (\beta_e e\partial_e + \beta_{m_H}m_H\partial_{m_H} + \beta_\xi\xi\partial_\xi)\hat{a}\right)w^{\text{gen}}(x) \\ [\tilde{\mathcal{C}}, \partial\frac{\delta}{\delta A}] &= \gamma_A\partial\frac{\delta}{\delta A} \end{aligned} \quad (5.28)$$

and with (5.26a)

$$\tilde{\mathcal{C}}\square B = \gamma_A\square B \quad (5.29)$$

Combining (5.28) and (29) with the local WI (5.27)

$$\left[\tilde{\mathcal{C}}, e\hat{a}w^{\text{gen}}(x) - \partial\frac{\delta}{\delta A}\right]\Gamma = \tilde{\mathcal{C}}\square B \quad (5.30)$$

we get the relation:

$$\gamma_A = \beta_e + (\beta_e e\partial_e + \beta_{m_H}m_H\partial_{m_H} - 2\gamma_A\xi\partial_\xi)\ln(1 + a) \quad (5.31)$$

Here we have also inserted equ. (5.26b). Since  $a$  is of order  $\hbar$ , the second term does not contribute in one-loop, so in one-loop we have

$$\gamma_A^{(1)} = \beta_e^{(1)}, \quad (5.32)$$

a relation which is also well-known from the unbroken version of the model. Higher orders are then recursively determined.

In (5.25) we have given the CS-equation in a manifestly  $W$ -symmetric form using the symmetrized operator  $\mathcal{N}_I$  and  $\tilde{\nabla}$ . For calculations, as for example the derivation of the leading logarithm behaviour, it is much more convenient to rewrite it into the usual form, which separates the hard and soft breaking on the left and right hand

side of the CS-equation. Summarizing thereby also the relations we have derived in (5.26) and (5.31) we end up with the following form:

$$\begin{aligned}
\mathcal{C}\Gamma &\equiv \left( \underline{m}\partial_{\underline{m}} + \beta_e e \partial_e + \beta_{m_H} m_H \partial_{m_H} - \gamma_s N_s - \hat{\gamma}_s \hat{N}_s - \bar{\gamma}_s \bar{N}_s \right. \\
&\quad \left. - \gamma_1 \int (\varphi_1 \frac{\delta}{\delta \varphi_1} - Y_1 \frac{\delta}{\delta Y_1} + \hat{\varphi}_1 \frac{\delta}{\delta \hat{\varphi}_1} + q_1 \frac{\delta}{\delta q_1}) - \gamma_A (N_A - N_B + 2\xi \partial_\xi) \right) \Gamma \\
&= -\frac{m}{e} \left( (\hat{\xi}_A + \alpha_1) \int \frac{\delta}{\delta \varphi_1} + (\xi_A + \hat{\alpha}_1) \int \frac{\delta}{\delta \hat{\varphi}_1} - \alpha_{\text{inv}} \int \frac{\delta}{\delta \varphi_0} \right) \Gamma \\
&\quad + \bar{\gamma}_s \int (q_1 Y_1 + q_2 Y_2)
\end{aligned} \tag{5.33}$$

with

$$\begin{aligned}
\gamma_1 &= \frac{1}{2} (\beta_e e \partial_e + \beta_{m_H} m_H \partial_{m_H} - 2\gamma_A \xi \partial_\xi) \ln \frac{z_1}{z_2} = O(\hbar^2) \\
\hat{\gamma}_s &= \beta_e + \gamma_A - \gamma_s + \gamma_1 \\
\gamma_A &= \beta_e + (\beta_e e \partial_e + \beta_{m_H} m_H \partial_{m_H} - 2\gamma_A \xi \partial_\xi) \ln(1+a) \\
\alpha_1 &= \left( (\gamma_s \hat{\xi}_A + \gamma_1 \hat{\xi}_A - \beta_e \hat{\xi}_A + \bar{\gamma}_s \xi_A) \right. \\
&\quad \left. + (\beta_e e \partial_e + \beta_{m_H} m_H \partial_{m_H} - 2\gamma_A \xi \partial_\xi) \hat{\xi}_A \right) \\
\hat{\alpha}_1 &= \left( (\hat{\gamma}_s \xi_A + \gamma_1 \xi_A - \beta_e \xi_A) \right. \\
&\quad \left. + (\beta_e e \partial_e + \beta_{m_H} m_H \partial_{m_H} - 2\gamma_A \xi \partial_\xi) \xi_A \right)
\end{aligned} \tag{5.33a}$$

The independent parameters are therefore the coefficient functions  $\beta_e, \beta_{m_H}, \gamma_s$  and  $\bar{\gamma}_s$ , which we give in the one-loop order in the next section, and the coefficient of the soft insertion  $\alpha_{\text{inv}} = \frac{1}{2} e \frac{m_H^2}{m} + O(\hbar)$ . Whereas in the one-loop order the hard anomalies are independent of the normalization conditions we have chosen, i.e. especially of  $\frac{z_1}{z_2}$ , one immediately verifies that one finds corrections due to the deformation of the rigid WI starting from two loop onwards. Let us emphasize again that the identification of the soft terms has been accomplished by use of the rigid symmetry.

## 6. Determination of coefficient functions

In the preceding sections we have derived Ward identities and the CS equ. in a way which is essentially independent of the scheme with which one regularizes and renormalizes the theory. To be complete and to come closer to practical work we determine the 1-loop coefficient functions of the CS-equation (5.33). Furthermore we want to demonstrate that the deformation coefficient of the Ward-identity is indeed non-trivial in the on-shell scheme.

### 6.1. DEFORMATION COEFFICIENT OF THE RIGID WI

In sect. 3 we proved the WI (3.44, 45) in which the coefficient  $z_1/z_2$  indicates the possible deformation of the classical approximation due to the normalization conditions (2.18d, e).

$$\begin{aligned} W^{\text{gen}}\Gamma & \equiv \int \left( -\sqrt{\frac{z_2}{z_1}}\varphi_2 \frac{\delta}{\delta\varphi_1} + \sqrt{\frac{z_1}{z_2}}(\varphi_1 - \hat{\xi}_A \frac{m}{e}) \frac{\delta}{\delta\varphi_2} - \sqrt{\frac{z_1}{z_2}}Y_2 \frac{\delta}{\delta Y_1} + \sqrt{\frac{z_2}{z_1}}Y_1 \frac{\delta}{\delta Y_2} \right. \\ & \quad \left. - \sqrt{\frac{z_2}{z_1}}\hat{\varphi}_2 \frac{\delta}{\delta\hat{\varphi}_1} + \sqrt{\frac{z_1}{z_2}}(\hat{\varphi}_1 - \xi_A \frac{m}{e}) \frac{\delta}{\delta\hat{\varphi}_2} - \sqrt{\frac{z_2}{z_1}}q_2 \frac{\delta}{\delta q_1} + \sqrt{\frac{z_1}{z_2}}q_1 \frac{\delta}{\delta q_2} \right) \Gamma = 0 \end{aligned} \quad (6.1)$$

We want to show in the following, that this coefficient is indeed non-trivial, i.e.  $\frac{z_1}{z_2} \neq 1$ , and gets higher order corrections unless one chooses some special unphysical normalization conditions.

The factor  $\frac{z_1}{z_2}$  can be determined by testing the WI with respect to  $\varphi_1$  and  $\varphi_2$ , it yields in momentum space

$$\begin{aligned} & -\sqrt{\frac{z_2}{z_1}}\Gamma_{\varphi_1\varphi_1}(p^2) + \sqrt{\frac{z_1}{z_2}}\Gamma_{\varphi_2\varphi_2}(p^2) \\ & -\sqrt{\frac{z_1}{z_2}}\frac{m}{e}(\hat{\xi}_A\Gamma_{\varphi_1\varphi_2\varphi_2}(p, -p, 0) + \xi_A\Gamma_{\varphi_1\varphi_2\hat{\varphi}_2}(p, -p, 0)) = 0 \end{aligned} \quad (6.2)$$

In order to project out the residua we differentiate with respect to the momentum  $p^2$  and get the following equation for the 1-loop coefficient  $u^{(1)}$  defined by  $\frac{z_1}{z_2} = 1 + u$  (3.46):

$$\begin{aligned} & \frac{1}{2}u^{(1)} - \partial_{p^2}\Gamma_{\varphi_1\varphi_1}^{(1)}(p^2) + \frac{1}{2}u^{(1)} + \partial_{p^2}\Gamma_{\varphi_2\varphi_2}^{(1)}(p^2) \\ & + \frac{m}{e}\partial_{p^2}\left((1 - x\xi_A)\Gamma_{\varphi_1\varphi_2\varphi_2}^{(1)}(p^2) - \xi_A\Gamma_{\varphi_1\varphi_2\hat{\varphi}_2}^{(1)}(p^2)\right) = 0 \end{aligned} \quad (6.3)$$

To determine  $u^{(1)}$  explicitly one had to calculate the 2-point and 3-point function according to (6.3), but we shall now show by simple arguments that  $u^{(1)}$  depends

only on the wave function normalization and turns out to differ from zero within physical normalization schemes.

For this purpose we use the normalization conditions (2.18d,e) in a slightly generalized form:

$$\begin{aligned}\partial_{p^2}\Gamma_{\varphi_2\varphi_2}(p^2 = \kappa^2) &= 1 \\ \partial_{p^2}\Gamma_{\varphi_1\varphi_1}(p^2 = \mu^2) &= 1\end{aligned}\tag{6.4}$$

where  $\mu$  is a further independent normalization point. According to (6.4) we can rewrite  $\partial_{p^2}\Gamma_{\varphi_i\varphi_i}$  into

$$\begin{aligned}\partial_{p^2}\Gamma_{\varphi_1\varphi_1} &= 1 + \partial_{p^2}\Sigma_1^{(1)}(p^2, m^2, m_H^2) - \partial_{p^2}\Sigma_1^{(1)}(p^2, m^2, m_H^2)\Big|_{p^2=\mu^2} + O(\hbar^2) \\ \partial_{p^2}\Gamma_{\varphi_2\varphi_2} &= 1 + \partial_{p^2}\Sigma_2^{(1)}(p^2, m^2, m_H^2) - \partial_{p^2}\Sigma_2^{(1)}(p^2, m^2, m_H^2)\Big|_{p^2=\kappa^2} + O(\hbar^2)\end{aligned}\tag{6.5}$$

$\Sigma_i^{(1)}(p^2, m^2, m_H^2)$  is the usual self energy calculated in a specific scheme as e.g. in the  $\overline{MS}$ -scheme.

Since equ. (6.3) is true for all momenta  $p^2$ , the momentum dependence has to cancel, and it can therefore be evaluated at every convenient value. One possibility is at a momentum large compared to the masses: There the three-point functions vanish asymptotically and according to the CS-equation one finds for the self energy contribution (see (6.10))

$$\lim_{p^2 \rightarrow -\infty} \partial_{p^2}\Sigma_1^{(1)}(p^2, m^2, m_H^2) = \lim_{p^2 \rightarrow -\infty} \partial_{p^2}\Sigma_2^{(1)}(p^2, m^2, m_H^2) \rightarrow -\gamma_s^{(1)} \ln \frac{p^2}{m^2}, \tag{6.6}$$

where we have normalized  $\partial_{p^2}\Sigma_i$  appropriately in the asymptotics because it is determined only up to constants. Therefore, in order to determine  $u^{(1)}$  one remains with

$$u^{(1)} + \partial_{p^2}\Sigma_1^{(1)}(p^2, m^2, m_H^2)\Big|_{p^2=\mu^2} - \partial_{p^2}\Sigma_2^{(1)}(p^2, m^2, m_H^2)\Big|_{p^2=\kappa^2} = 0 \tag{6.7}$$

Inspection of the diagrams shows that  $\Sigma_1^{(1)}$  and  $\Sigma_2^{(1)}$  differ at least by contributions built up from diagrams with trilinear  $\varphi$ -vertices. To be more specific we fix the residuum of the unphysical particle  $\varphi_2$  at a momentum  $\kappa$  large compared to the masses, i.e.

$$\partial_{p^2}\Gamma_{\varphi_2\varphi_2}(p^2 = \kappa^2, |\kappa^2| \gg m^2, m_H^2) = 1 \tag{6.8}$$

Such normalization conditions are e.g. implicitly chosen if one calculates the self energy of  $\varphi_2$  in the  $\overline{MS}$ -scheme. With (6.8) equ. (6.7) simplifies to

$$u^{(1)} + \partial_{p^2} \Sigma_1^{(1)}(p^2, m^2, m_H^2) \Big|_{p^2=\mu^2} + \gamma_s^{(1)} \ln \frac{\kappa^2}{m^2} = 0 \quad (6.9)$$

Hence  $u^{(1)}$  will differ from zero, unless we fix the wave function renormalizations *both* at the same asymptotic momentum,  $\mu^2 = \kappa^2$ . Such unphysical normalization conditions are not useful if one wants to calculate the  $S$ -matrix, because there the wave function normalization of the physical particle at finite momentum is needed. We conclude that otherwise, especially within the physical on-shell scheme  $\mu^2 = m_H^2$  (2.18), the WI-operator is indeed deformed by  $u_1^{(1)} \neq 0$ .

## 6.2. COEFFICIENTS OF THE CS-EQUATION

Due to the inclusion of the external field multiplet  $\hat{\varphi}$  and the parametrization as arising from physical normalization conditions the CS-equ. (5.33) has a slightly unconventional form. We therefore calculate the independent 1-loop coefficients explicitly.

Let us first determine  $\gamma_s$ . It is found by studying the action of the CS operator  $\mathcal{C}$  (5.33) on  $\partial_{p^2} \Gamma_{\varphi_1 \varphi_1}$ . Since the CS-equ. is valid for all momenta we can go to infinite momentum, where the r.h.s., the soft insertions, vanish. Therefore we get

$$\begin{aligned} \underline{m} \partial_{\underline{m}} \partial_{p^2} \Gamma_{\varphi_1 \varphi_1}^{(1)} - 2\gamma_s^{(1)} \partial_{p^2} \Gamma_{\varphi_1 \varphi_1}^{(o)} = \\ \underline{m} \partial_{\underline{m}} \partial_{p^2} \Gamma_{\varphi_1 \varphi_1}^{(1)} - 2\gamma_s^{(1)} p^2 \xrightarrow{-\infty} 0 \end{aligned} \quad (6.10)$$

Hence we are left with the calculation of  $\partial_{p^2} \Gamma_{\varphi_1 \varphi_1}$  at asymptotic momentum. This yields

$$\gamma_s^{(1)} = -\frac{e^2}{16\pi^2} (3 - \xi) \quad (6.11)$$

Next we calculate  $\beta_e^{(1)}$ . According to (5.32) this simplifies to the calculation of  $\gamma_A^{(1)}$ , given by the vector self-energy. For asymptotic momentum analogous arguments are valid for the disappearance of three-point contributions and the evaluation of the two-point functions leads to

$$\gamma_A^{(1)} = \frac{1}{16\pi^2} \cdot \frac{1}{3} e^2 = \frac{1}{e} \beta_e^{(1)}, \quad (6.12)$$

For  $\beta_{m_H}$  the relevant vertex function is the four-point function  $\Gamma_{\varphi_1\varphi_1\varphi_1\varphi_1}$ :

$$\underline{m}\partial_{\underline{m}}\Gamma_{\varphi_1\varphi_1\varphi_1\varphi_1}^{(1)} - 6\beta_e^{(1)}e^2\frac{m_H^2}{m^2} - 6\beta_{m_H}^{(1)}e^2\frac{m_H^2}{m^2} + 12\gamma_s^{(1)}\frac{m_H^2}{m^2}p^2 \xrightarrow{p^2 \rightarrow -\infty} 0 \quad (6.13)$$

and one gets

$$\beta_{m_H}^{(1)} = \frac{e^2}{16\pi^2} \left( 5\frac{m_H^2}{m^2} + 6\frac{m^2}{m_H^2} - \frac{19}{3} \right) \quad (6.14)$$

The functions  $\gamma_s^{(1)}, \beta_e^{(1)}$  and  $\beta_{m_H}^{(1)}$  are determined by the vertex functions of the quantum fields and are in agreement with the symmetric theory.

The coefficient function  $\bar{\gamma}_S$  is determined by vertex functions including external fields. It can be found by acting with  $\mathcal{C}$  on  $\partial_{p^2}\Gamma_{\varphi_1\hat{\varphi}_1}$ . Again taking the limit of infinite  $p^2$  we find

$$\underline{m}\partial_{\underline{m}}\partial_{p^2}\Gamma_{\varphi_1\hat{\varphi}_1}^{(1)} - \gamma_s^{(1)}x - \bar{\gamma}_s^{(1)} - (2\frac{\beta_e^{(1)}}{e} - \gamma_s^{(1)})x \xrightarrow{p^2 \rightarrow -\infty} 0 \quad (6.15)$$

(we have used (5.26c)). The evaluation of the respective diagrams leads eventually to the result

$$\bar{\gamma}_s^{(1)} = \frac{2}{16\pi^2}e^2 + 2\left(\gamma_s^{(1)} - \frac{\beta_e^{(1)}}{e}\right)x \quad (6.16)$$

One has to note, that  $\bar{\gamma}_s$  depends on the parameter  $x$  and is different from zero, even if we take  $x = 0$ .

## 7. Discussion and conclusions

In models with gauge invariance the rigid invariance encodes the physical consequences of the symmetry: e.g. the conservation of quantum numbers in physical processes as the consequence of conserved charges; the arrangement of particles in multiplets; definite relations amongst physical amplitudes. In case that the symmetry is not broken it can be implemented easily in every renormalization scheme and requires no special care. Whether (formal) unitarity is guaranteed by a local WI (in the abelian case) or by the ST identity (in the non-abelian case) does not really matter, the rigid WI can just be written down naively. In the case of spontaneous breakdown of the rigid symmetry the situation changes. A conserved charge does no longer exist, the consequences of the symmetry reside entirely in relations amongst

amplitudes, which have to be deduced from a WI. Unitarity (formal or full) can only be deduced from the ST identity which also serves as the unique characterization of the model. The WI for the rigid symmetry is no longer trivial to deduce, i.e. one has to organize radiative corrections with quite some care. It has to be formulated explicitly in accordance with the ST identity and its form turns out to depend on the normalization conditions. In the abelian Higgs model which we treated in the present paper we have seen that higher orders deform the WI in a well specified way once we stick to physical on-shell normalization conditions. Hence the relations amongst Green functions found in lowest order as a consequence of the WI are deformed very specifically in higher orders. The ST identity guarantees unitarity and restricts the rigid WI, but does not uniquely fix it. Once the rigid WI has been constructed it is immediately useful for the formulation of the CS equ. It would be quite difficult to handle there the soft terms i.e. to define the higher orders without the rigid invariance. In more complicated theories like the standard model it is virtually impossible to proceed without it. There a deformation of the classical WI operator not only involves relative factors for the fields within one multiplet, but also for the several non-abelian generators relative to each other, in particular the orientation of the electromagnetic  $U(1)$  relative to the remainder of the group. The details will be reported elsewhere.

As an important technical tool – again indispensable in the standard model – we have introduced a doublet of external scalar fields in order to formulate the 't Hooft gauge and its breaking of the rigid symmetry in a controllable way. They render the rigid WI homogeneous and thus manageable in higher orders. How they form the building block to a background field formulation of the model remains to be explored.

*Acknowledgement* We are grateful to B. Kniehl for drawing our attention to the references [2, 3].



## Appendix    The Propagators

The bilinear part of the classical action reads

$$\begin{aligned}\Gamma_{\text{bil.}} = & \int \left( -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \right. \\ & + \frac{1}{2}(\partial\varphi_1\partial\varphi_1 + \partial\varphi_2\partial\varphi_2) - m\partial\varphi_2 A + \frac{1}{2}m^2 A^2 - \frac{1}{2}m_H^2\varphi_1^2 \\ & \left. + \frac{1}{2}\xi B^2 + B(\partial A + \xi_A m\varphi_2) - \bar{c}\square c - \xi_A m^2 \bar{c}c \right)\end{aligned}$$

It gives rise to the following propagators:

$$\begin{aligned}G_{\varphi_1\varphi_1}(p, -p) &= \frac{i}{p^2 - m_H^2} \\ G_{BB}(p, -p) &= 0 \\ G_{BA_\mu}(p, -p) &= \frac{-p^\mu}{p^2 - \xi_A m^2} \\ G_{B\varphi_2}(p, -p) &= \frac{-im}{p^2 - \xi_A m^2} \\ G_{\varphi_2\varphi_2}(p, -p) &= i \frac{p^2 - \xi m^2}{(p^2 - \xi_A m^2)^2} = i \left( \frac{1}{p^2 - \xi_A m^2} + \frac{m^2(\xi_A - \xi)}{(p^2 - \xi_A m^2)^2} \right) \\ G_{\varphi_2 A_\mu}(p, -p) &= \frac{-m(\xi - \xi_A)p_\mu}{(p^2 - \xi_A m^2)^2} \\ G_{A_\mu A_\nu}(p, -p) &= \left( \eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) G^T + \frac{p^\mu p^\nu}{p^2} G^L \\ G^T(p^2) &= \frac{-i}{p^2 - m^2} \\ G^L(p^2) &= i \left( \frac{-\xi}{p^2 - \xi_A m^2} + \frac{(\xi_A - \xi)\xi_A m^2}{(p^2 - \xi_A m^2)^2} \right)\end{aligned}$$

For the Fourier transformation we have used the conventions:

$$G_{\phi_a\phi_b}(x, y) = \int \frac{d^4p}{(2\pi)^4} G_{\phi_a\phi_b}(p, -p) e^{ip(x-y)}$$

## REFERENCES

- [1] B. A. KNiehl, *Status of higher-order corrections in the standard electroweak theory*, KEK-TH-412, Sept. 1994
- [2] R. BARBIERI, M. BECCARIA, P. CIAFALONI, G. CURCI, A. VICERÉ, *Phys. Lett.* **B288** (1992) 95
- [3] G. DEGRASSI, S. FANCHIOTTI, P. GAMBINO, *Two-loop next-to-leading  $m_t$  corrections to the  $\rho$  parameter*, CERN-TH.7180/94
- [4] K.I. AOKI, Z. HIOKI, R. KAWABE, M. KONUMA, T. MUTA, *Suppl. Prog. Theor. Phys.* **73** (1982) 1
- [5] M. BÖHM, W. HOLLIK, H. SPIESBERGER, *Fortschr. Phys.* **34** (1986) 687
- [6] F. JEGERLEHNER, *Renormalizing the standard model*, in: Proceedings of the 1990 Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, ed. M. Cvetič and P. Langacker, 1991, Singapore
- [7] C. BECCHI, A. ROUET, R. STORA, *Comm. Math. Phys.* **42** (1975) 127
- [8] G. SHORE, *Ann. of Phys.* **137** (1981) 262
- [9] M.B. EINHORN, J. WUDKA, *Phys. Rev.* **D39** (1989) 2758
- [10] A. DENNER, S. DITTMAIER, G. WEIGLEIN, *Nucl. Phys.B, Proc. Suppl.* **37B** (1994) 87
- [11] E. KRAUS, *The structure of the invariant charge in massive theories with one coupling*, BUTP 93-26, to be published in *Ann. of Phys.*
- [12] E. KRAUS, *Helv. Phys. Acta* **67** (1994) 424